Parallel algorithm of spline-based wavelet transform

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Abstract. We consider the issue of loosening heavy-tails in process of analyzing data in queueing theory with using spline wavelet functions. Algorithms of decomposition and reconstruction of sequences of random variables were shown. Options of algorithms paralleling, their advantages and disadvantages were assessed. We research and analyze the results of computational experiment on fixed traffics.

Keywords: spline wavelets, discrete wavelet transform, parallel algorithm.

1. Introduction

Spline wavelet functions are being increasingly used in different fields of science during recent decades. Scientists of the whole world are developing wavelet theory for solving different applied tasks. Discovery of wavelets is a big leap forward in the development of image and signal processing theory. It is because wavelet functions root in highly effective tools for working with local singularities of functions, unlike the traditional apparatus.

It is important to choose appropriate basis when we are building wavelets. Many researches are devoted to this issue [1,2]. In this report we are considering wavelets based on spline functions. Modern Russian [3,4] and foreign [5] scientists are thoroughly studying this type of wavelets, and is most efficient for such task as loosening heavy tails. As the main application of algorithms of decomposition and reconstruction of data sequences is focused on working with large amounts of data, so speed of processing this streams becomes very important, along with the quality of this process.

In this report we are showing parallel algorithms for direct and back spline wavelet transformation. Parallelization of these algorithms was made for dual-processor architecture. Efficiency of using these algorithms in comparison with serial implementation is especially evident in large sequences of data.

2. Loosening of heavy-tails in the analysis of traffic in queuing theory

2.1. Formulation of the problem

Researches are showing that traffic has self-similar properties. This is expressed in strong correlation of sequences of random variables. One of the signs of self-similarity is availability of heavy tails, i.e. large redundancy of appropriate integral functions of distribution, for example, Pareto distribution, Log-normal distribution and others. Analysis of correlated traffic by classical methods of queueing theory is impossible. This is why there arise the issues of decorrelation of traffic sampling rate for further analysis.

2.2. General scheme of the solution
We can apply different orthogonal transformations for loosening the correlation sequence of strong correlation random variables, for example, distribution by Karhunen-Loeve, Fourier and others. But different traffic sampling rates can have different local singularities of a signal. Wavelet functions are suitable for this goal. General scheme of solving this problem is decomposition of sequence of data, which characterize the traffic, using wavelet transformation. In this report we consider discrete wavelet transformation (DWT), based on splines.

3. Spline wavelet functions

3.1. General scheme of the construction
We consider space of polynomial splines with degree $m-1$ and power 1 $L_p = S(\Delta_p, m-1, 1)$ in each segments Δ_p of the initial segment $[0, n]$. For each $p \geq n_0$, when n_0 - integer, that $2^{n_0} \leq 2m-1 \leq 2^{n_0+1}$, the space $S(\Delta_p, m-1, 1)$ we can imagine as the direct sum $L_p = L_{n_0} \oplus W_{n_0+1} \oplus W_{n_0+2} \oplus ... \oplus W_p$, when W_p is orthogonal in terms of $L_2[0,n]$ addition of the space L_{p-1} to the space L_p. Wanted basis is got with union basis in L_{n_0} and all other basis in spaces W_p, $n_0 < p \leq k$. A detailed algorithm for constructing a spline wavelet basis is described in [6].

3.2. Scheme of algorithms of decomposition and reconstruction
Algorithm of decomposition of initial sequence of data is consists in iteration finding coefficients d and c through wavelet functions and B-splines. Every next coefficient is calculated through a previous.

$$\begin{tikzcd}
\tilde{d}_k \arrow[r] \arrow[dr] & \tilde{d}_{k-1} \arrow[r] \arrow[dr] & \tilde{d}_{k-2} \arrow[r] \arrow[dr] & \cdots \arrow[r] \arrow[dr] & \tilde{d}_{n_0} \\
& \check{c}_k & \check{c}_{k-1} & \cdots & \check{c}_{n_0+1}
\end{tikzcd}$$

Spline wavelets are semi-orthogonal, so for finding coefficients, we should solve set of $k-n_0+1$ independent systems of linear equations:

$$\left(\sum_{j=-m+1}^{2^{n_0}-1} d_{0j}\phi_{j,n_0}, \phi_{l,n_0}\right) = (f, \phi_{l,n_0}), -m+1 \leq l \leq 2^{n_0}-1 \tag{1}$$

$$\left(\sum_{j=-m+1}^{2^{n_0+p}-m} c_{pj}\psi_{j,n_0}, \psi_{l,n_0}\right) = (f, \psi_{l,n_0+p}), 1 \leq p \leq k-n_0, -m+1 \leq l \leq 2^{n_0+p}-m \tag{2}$$

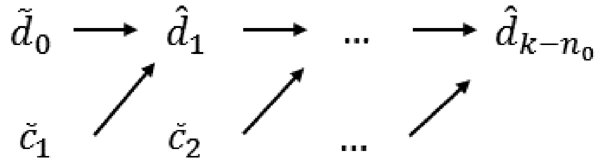
, when $\psi_{i,n} = \sum_{j=2i}^{2i+3m-2} \alpha_j \phi_{j,n-1}$ - wavelet function, and $\phi_{j,n}$ - normalized B-spline. The fast algorithm exists for calculating all scalar products in the right part of the systems (1,2). This follows from spline wavelet properties:

$$d_k = (f, \phi_{i,k}) = \sum_{j=2i}^{2i+3m-2} \beta_{kj} d_{j,k+1} \tag{3}$$

$$c_k = (f, \psi_{i,k}) = \begin{cases} \sum_{j=2k}^{2k+3m-2} \alpha_{kj} d_{j,k}, 1 \leq k \leq 2^{n-1}-2m+1 \\ \sum_{j=2k}^{2^n-1} \alpha_{kj} d_{j,k}, 2^{n-1}-2m+2 \leq k \leq 2^{n-1}-m \\ \sum_{j=-m+1}^{2k+3m-2} \alpha_{kj} d_{j,k}, -m+1 \leq k \leq -1 \end{cases} \tag{4}$$

Please note the following. The initial vector of random variables which characterizes traffic, is divided into a set of vectors $\left(2^k+1\right)$ -elements each. Each of them is processed independently by systems (1,2). As a result we get the desired decorrelation sequence of data.

Reconstruction of the initial vector of data is finding of all values of the set through known coefficients d and c.



The formula for calculating one layer of coefficients is:

$$d_{i+1} = \tilde{d}_i + \tilde{d}_i \tag{5}$$

when

$$\tilde{d}_i = \sum_j d_{ij}\beta_j, \tilde{c}_i = \sum_j c_{ij}\alpha_j \tag{6}$$

Then we can reconstruction values of the initial set by formula

$$f(x_{ij}) = \sum_{s=-m+1}^{2^k-1} d_{k-n_0,s}\phi_{k,s}(x_{ij}) \tag{7}$$

Here α, β - coefficients are calculated by special algorithm [6]. For example, for $m = 2$ $\alpha = (1, -6, 10, -6, 1), \beta = (1, 2, 1)$.

Algorithms of decomposition and reconstruction are described in detail in [6].

4. Steps of parallelization of serial algorithms of the spline DWT

4.1. Test infrastructure

For experiments, we have fixed traffic as sequences, which characterize time between neighboring packages in each unit of cloud infrastructure. It was deployed specially for test infrastructure.

Numerical experiment was conducted with using: programming language - C++, development environment - Microsoft Visual Studio 15.2, compiler - Microsoft C/C++ Compiler Version 19.10.25019, CPU - Quad-core processor Intel Core i5-3470 CPU 3.20 GHz, Open Multi-Processing API.

The sizes of samples are from 10 000 to 300 000 elements.

4.2. Parallelization of decomposition and reconstruction algorithms

We note that calculating coefficients of initial sequence of data execute independently for each layer of its dividing into vectors with $\left(2^k+1\right)$ elements, by solving systems (1,2). Accordingly, we can execute parallelization of serial implementation of decomposition algorithms on a higher level. We can render a set of systems of every layer for each stream, so that the total number of operations will be approximately equal. This approach provides uniformity of load distribution between the streams.

Similarly, we can execute parallelization of reconstruction algorithm of vector to the initial signal. Formula (7) is applied to each layer of dividing in $\left(2^k+1\right)$ elements for transformed sequences. Set of sums is sent to each stream with uniform load distribution.

Table 1 includes the results of computation experiments executed for serial and parallel implementations.

Table 1. Time of working reconstruction algorithms.

	Sample size	Serial impl. (sec)	Parallel impl. (sec)	Acceleration
1	10 000	0.016	0.015	0.017
2	100 000	0.055	0.031	1.774
3	300 000	0.148	0.070	2.114

5. Conclusion

In course of this work we have executed serial and parallel implementation of algorithms of spline DWT, which can loosen heavy ails for traffic analysis in queueing theory. Implementation of parallel scheme made it possible to reduce the time of calculating coefficients 1.9 times.

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7. References

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